Total integrals of solutions for inhomogeneous Painlevé II equation

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We establish a formula determining the value of the Cauchy integrals for the real and purely imaginary Ablowitz-Segur solutions for the inhomogeneous second Painlevé (PII) equation

$$u''(z) = zu(z) + 2u^3(z) - \alpha, \quad z \in \mathbb{C}, \tag{1}$$

where $\alpha \in \mathbb{C}$ is a constant. The formula generalizes the results of [1], where the values of the Cauchy integrals were derived for the Ablowitz-Segur solutions of the homogeneous PII equation ($\alpha = 0$). Our approach relies on the Deift-Zhou steepest descent analysis of the corresponding Riemann-Hilbert problem and the construction of an appropriate parametrix in a neighborhood of the origin. The obtained results are used to provide a rigorous proof of a numerically predicted phenomena that an arbitrary logarithmic spiral is a finite time singularity developed by a geometric flow, which approximates the vortex patch dynamics of the 2D Euler equation.

References

[1] J. Baik, R. Buckingham, J. DiFranco, A. Its, *Total integrals of global solutions to Painlevé II*, Nonlinearity **22** (2009), no. 5, 1021–1061.

[2] P. Kokocki, *Total integrals of solutions for the inhomogeneous Painlevé II equation*, arXiv:1808.09039, accepted to Studies in Applied Mathematics