Extremal Markovian sequences of the Kendall type

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We consider the following extremal Markovian sequence:

$$X_0 = 1, \quad X_1 = Y_1, \quad X_{n+1} = M_{n+1} \left[\mathbf{I}(\xi_n < \varrho_{n+1}) + \theta_{n+1} \mathbf{I}(\xi_n > \varrho_{n+1}) \right],$$

where

$$\begin{aligned} M_{n+1} &= \max\{|X_n|, |Y_{n+1}|\} \cdot \{sgn(r) : \max\{|X_n|, |Y_{n+1}|\} = |r|\}, \\ \varrho_{n+1} &= \frac{\min\{|X_n|, |Y_{n+1}|\}^\alpha}{\max\{|X_n|, |Y_{n+1}|\}^\alpha}, \end{aligned}$$

and

(i) $(Y_k) \sim i.i.d.(\nu)$,

(ii) $(\xi_k) \sim i.i.d.(U([0,1]))$,

(iii) $(\theta_k) \sim i.i.d.(\widetilde{\pi}_{2\alpha})$, $\widetilde{\pi}_{2\alpha}(dy) = \alpha|y|^{-2\alpha-1} \mathbf{1}_{[1,\infty)}(|y|)\, dy$,

(iv) (Y_k), (ξ_k) and (θ_k) are independent,

(v) θ_{n+1}, M_{n+1} are independent.

The stochastic process is the Markov process and the Lévy process in generalized convolution sense ([6]). Structure of considered processes is similar to the first order autoregressive maximal Pareto processes ([3], [4],[15]), the max-autoregressive moving average processes MARMA ([9]), minification processes ([14], [15]), extremal Markovian sequences ([1]), pARMAX and pRARMAX processes ([8]) or perpetuity.

Our construction is based on the Kendall convolution ([11]):

$$\delta_x \vartriangle_\alpha \delta_1 = |x|^\alpha \widetilde{\pi}_{2\alpha} + (1 - |x|^\alpha) \widetilde{\delta}_1, \quad x \in [-1, 1].$$

In other words, we live in the Kendall convolution algebra, where

$$\delta_1 \vartriangle_\alpha \delta_1 = \widetilde{\pi}_{2\alpha}.$$

Why probabilistic objects in the Kendall convolution algebra are important? Because distributions generated by the Kendall convolution have generally heavy tailed distributions ([2], [5], [7]), limit distributions belong to domain of attraction of the Fréchet distribution, so they have connections with the theory of extremes. Consequently, there is a possibility of use them for modeling certain extreme events such as indicators of air pollution and water levels.

We prove some properties of hitting times and an analogue of the Wiener-Hopf factorization for the Kendall random walk ([11], [12], [13]). We show also that the Williamson transform ([18]) is the best tool for problems connected with the Kendall generalized convolution.

Continuing the results obtained in [12] we construct renewal processes for extremal Markovian sequences of the Kendall type and present significant properties of Kendall random walks on the positive half line.

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