REPORT ON THE PHD THESIS OF G. JAGIELLA, AT UNIWERSYTET WROCLAWSKI

The PhD thesis of Grzegorz Jagiella investigates notions from topological dynamics, through the eyes of a model theorist, with the final aim of analyzing Lie groups, (mainly the example of semisimple groups) which are definable in o-minimal structures over the reals. While I am not an expert in topological dynamics, I was very impressed by the thesis, by the (surprising to me) level of technicalities that some of the results required and by the general framework which is laid out in the thesis before even getting to the final results about definable groups in o-minimal structures.

While the thesis might lack distinct main theorems it sets up the ground for future work on definable groups in o-minimal structures and possibly also in other tangential areas (thus, for example, Sections 3 is general and could possibly be useful for settings other than o-minimality).

I admit that I did not read in details some of the proofs, which I found to be technical and, for me, lacking intuition (e.g. 3.1.4, 4.3.10). Still, all together, the author demonstrates a combination of skills in various areas of mathematics such as Lie groups, topological dynamics and the model theoretic calculus of forking. Therefore, I find it to be **an excellent thesis** and I recommend considering it for one of the university prizes which were mentioned in the documents that I received.

Below ar some more detailed comments on the thesis. My most repeating "complaint" is about the choice of notation and about the fact that various proofs and discussions are too terse. I found very few actual gaps in proofs, so my impression is that the results are indeed true. Since some of the results were already published it might be not necessary to fix these (minor) flaws now , but for sake of completeness I include the comments.

Section 2 I found the introduction to be well written and useful, especially with regard to topological dynamics. One drawback is that it lacks references, which in a Ph.D. thesis one might expect.

Section 3 The underlying assumption that all types are definable should be given more emphasis. While reading the thesis I had to remind myself of this point several times.

p.16 In the second paragraph the author introduces the action of G on K. Isn't it just the same as the action on G/H, via its natural identification with K? Wouldn't it be simpler to view it in this way?

In line 4 of Proposition 3.1.4, should the first $G(M)$ be $K(M)$ (in the pair $(G(M), S_K(M))$?

Proposition 3.2.1 In the statement and the proof of the proposition, the use of $H(M)$ is unclear. Since this is a family of definable sets it makes more sense to denote it by $imd_q \cdot H$ and to use H everywhere rather than $H(M)$. In line 4 of the proof, a more precise reference for [23] is required.

Also, what is the role of 3.2.1 in this section? Isn't it out of place?

Lemma 3.2.2 Instead of assuming that all types are definable it is sufficient to assume that r is definable.

Lemma 3.2.3 Which action of H is considered here?

Top of p. 22 Should it be $h \models p|h'_1k$ (and not $h \models hp|h'k$)?

22^{10} I could not understand justification for the third $=$ in the sequence.

22_4 Why is $p *_1 (p *_1 r) = (p * p) *_1 r$?

Section 4

Proposition 4.0.5 in line 4, the "partial functions" should be bijections.

23_9 The assumptions on the underlying structures in various sections should be made clearer. In some other places (e.g. p. 25) M is a fixed o-minimal structure over the reals.

Definition 4.0.6 One usually says "over M" when referring to an underlying set of parameters. Thus "dimension over M" is better replaced by "dimension in M". Also, in clause (i), is there a reason to say "contains a block" rather than "contains an open set"? Also, is there a reason for the use of M in the notation $dim_M(-)$, rather than just $\dim(-)$?

24^4 What is a "non-degenerate" function?

24_2 The use of $\mathbb{R}$ for the set of reals as well as the o-minimal structure is very confusing and does not allow for example to expand the language on the same underlying set.

25^7 The use of "types over R^{ext}" is confusing. Types are usually 'over" a set of parameters, with some fixed language in mind. Did you possibly mean $\mathbb{R}^{ext}$?

25_9 Proposition 4.1.1 is a result of Miller and Starchenko and not as written. In any case, the result is true for all o-minimal structures, not only for elementary extensions of structures over the reals.

28^2 Lemma 4.2.2 is not true as stated, unless it is assumed that U is defined over $\mathbb{R}$ in which case there is no need to use R at all. In addition, the notation $\dim_{\mathbb{R}}(-)$ is ambiguous.

28^{11-14} It is not clear to me why the collection of formulas above can be extended to a generic type in $S_{ext,G(R)}$. Is there a general claim here that for G definably compact and $X \subseteq G(R)$ externally defined, either X or $G \setminus X$ are generic? Even if such extension exists, why is it unique? All these claims are sufficiently interesting to justify spending some more time on.

Proposition 4.2.3 What is the object Gen/G^{00}? Also, I had a hard time following the discussion at the bottom of p. 28. as several claims are described without an obvious explanation.

Also, I suggest to put the definition of $u(g)$ separately as the author keeps referring to it in the text.

29^5 This remark is not clear to me. Why doesn't $r \star r'$ depend on $st(r)$, $st(r')$?

30^{2-6} Presumably, it is meant here that K, H are themselves definable over $\mathbb{R}$ (again, the meaning of "over" here refers to the parameter set). The terms "types over R^{ext}" and "dimension over R^{ext}" are confusing.

30^9 The use of $G(R^{ext})$ is once again confusing. Why not use $G(R)$ here?

Proposition 4.3.4 and its proof Why is I unique? Also, I suggest to say explicitly what I is. What does "arbitrary generic" mean in the proof? In the very last sentence of this proof 3.1.4 is invoked but if so, then the assumptions should be verified in this R^{ext}-situation. E.g. why is $H(R^{ext})$ extremely definably amenable?

31_7 it seems as if a type p is fixed here (for the rest of this section?). This needs to be made precise as I found myself struggling with this p throughout the reading.

Proof of 4.3.7 I could not understand this proof. E.g. why is there such a set V (and don't you need also that $f(st(b)) \notin Cl(V)$? At the very end of the proof, why does the existence of V' yield a contradiction?

Proof of 4.3.8 The last statement about the set coinciding with the graph of ψ, needs an explanation.

Proof 4.3.10 Although I did not read this proof in details I did not see why h realizes a weakly generic type.

Corollary 4.3.11 Where does g vary in this statement?

34^{2-5} What is the role of these two corollaries here? In what sense these corollaries of the previous results in this section?

Proposition 4.4.1 Presumably, the word "algebraic" should be replaced by "abstract"

35^{12-13} The statement about the induced co-cycle on the type space requires a proof (and as before, could be interesting on its own right, thus deserves some space).

Example 4.4.7 What is the reference for the fact that the Ellis subgroup is the pro-finite completion of $\mathbb{Z}$?

Example 4.4.8 I could not make sense of the sentence starting with "the $\mathcal{M}$-structure on ....".