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Report on the doctoral thesis "Gromov positive scalar curvature conjecture and rationally inessential macroscopically large manifolds. Tilings of manifolds and uniformly bounded homology" by Michal Marcinkowski

Dear Prof Debicki,

here is my report on the thesis of Michal Marcinkowksi.

Inspired by its geometric relevance, recent decades have seen a flurry of research activity around the question which closed manifolds admit a Riemannian metric of positive scalar curvature.

A (somewhat vauge) insight of Mikhael Gromov, one of leaders of the field, is that the main reason for positive scalar curvature should be that the manifold has at least two directions into which it is "rolled up". The prime example of this phenomenon are manifolds M of the form $M = X \times S^k$ for $k \geq 2$: these all admit metrics of positive scalar curvature and the S^k-factor is "rolled up".

More precisely, this idea of "being rolled up" is described by the fact that also the universal covering $\tilde{M}$ is small in at least 2 directions.

The mathematically precise notion here is the notion of "macroscopic dimension", defined in terms of the existence of surjective maps to d-dimensional simplicial complexes (such that the diameters of point-inverses have uniformly bounded diameter).

Then Gromov's macroscopic curvature conjecture states that an n-dimensional manifold with positive scalar curvature will have a universal covering $\tilde{M}$ whose macroscopic dimension is at most $n-2$. A weakening of this would say that the bound is $n-1$.

Dranishnikov developped a homological criterion for macroscopic dimension, using a certain adapted geometric homology. However, one might hope that one can avoid this complication and work with standard homological methods to determine the macroscopic dimension of a manifold M.

Indeed, an oriented n-dimensional manifold M is *rationally inessential* if the image $u_*[M] \in H_n(B\pi_1 M; Q)$ of its fundamental class under the map induced by the classifying map $u\colon M \to B\pi_1 M$ of its universal covering is zero.

Dranishnikov conjectured that rationally inessential manifolds have macroscopic dimension $\leq n-1$. This would prove the weak form of Gromov's conjecture for rationally inessential manifolds.

The main result of the thesis is the construction of counterexamples to Dranishnikov's conjecture.

This is achieved with a very clever construction, making use of a general machine for the construction of manifolds with specific homological properties due to Mike Davis. The construction is a clever use of the "avis reflection trick" construction, or rather a variant of it. It has to be carried out in such a way as to control also the exotic homology theory of Dranishnikov which governs macroscopic dimension (and the usual homology as well).

I do think that this is an important result, and by far not obvious. The significance is as follows: there is a very clear question (of Dranishnikov) around a significant geometric idea due to Gromov; and with a clever construction the thesis completely answers the question (negatively). I'm sure that it can be published in a leading international mathematical journal.

The author then also explores to which extend the construction can shed light on Gromov's original conjecture. He establishes Gromov's strong conjecture for a large subclass of the examples he constructs.

As an "add on", the thesis treats a second, completely unrelated, problem: the construction of aperiodic tilings for certain non-compact Riemannian manifolds on which certain amenable groups act cocompactly and isometrically. Such tilings had been constructed previously by Block and Weinberger, but only using highly non-amenable group actions. The main point here is the identification of appropriate ways to obtain such tilings (which follows Block-Weinberger, but using finite coefficients). The trick which makes the tiling aperiodic is the identification of a group theoretic structure which says that the symmetries necessary to make the tiling periodic simply don't exist. A drastic way to impose this would be to look only at groups which have no finite quotient group (leading to automatic aperiodicity). Here, a much weaker requirement (focusing on a single prime p) is identified, which is satisfied in intersting cases. This part of the thesis is a nice result, but probably of less significance than the main result discussed above.

The thesis is generally well written and correct. In a couple of places I would have liked to see a bit more detail and background (certainly possible as the thesis still is relatively short).

Alltogether, it is an important contribution and does fullfill all the requirements to be accepted as a doctoral thesis.

Therefore, I recommend to accept this doctoral thesis.

Göttingen, July 14, 2015

(Thomas Schick)

Following are some details concerning this, which the author might want to implement also in publications which might grow out of the thesis.

- p5, after "Gromov conjecture". There is a reference to Lemma ??
- p5 l-4: I'm not sure whether the "Gromov-Lawson conjecture" (in its original form) was only for aspherical manifolds
- Example 2: this is not quite to the point, as macroscopic dimension perhaps should only be considered for (universal?) coverings of compact spaces, and if one focuses on non-compact metric spaces probably they should be complete. The covering is *not* the universal covering, which probably should be pointed out (that would change if one started with T^3).
- Section 1.2.2: Start/Definition of locally finite homology: either X is a simplicial complex (to work with simplicial chains) or a CW-complex, then one has to work with cellular chains.
- p9 top: the map "ec_*" denoted "equivariant coarsening map" is usually called "transfer" and is a standard tool in algebraic topology: why introduce new notation?
- Theorem 1: should also work if each skeleton of $B\Gamma$ is finite; otherwise torsion in the fundamental group is not allowed.
  in the formulation of Theorem 1, "if an only" should be "if and only"
- Proof of Corollary 1, last line: "pass an orientable double cover" should be "pass to an orientable double cover"; but I would feel that one should say a sentence more (like "the universal cover is unchanged, therefore...")
- p 10 middle: in the construction of M_L, the elements of the vector space G are multiplied; which is unusual notation; also point out that only points in the basis of the cone are multiplied
- p10 l-6: "versors" should be "vectors"
- p11 l-4: stress that the existence of σ such that the finite order element g can be written in terms of these special generators is a consequence of the known strong results on the structure of Coxeter groups and its elements
- p12, before Corollary 2: Here, cones are defined, but they have been used many times before in the thesis; this order does not make much sense
- Example 3, l2-3: "symplex" should be "simplex" and "is draw" should be "is drawn"
- Proof of Lemma 5: discuss the issue of orientation: the framings have to be chosen appropriately to preserve orientability. The sentence "The homology class of the image does not change" does not really make sense to me. He probably wants to say that the image of the fundamental class in the homology of X is unchanged (provided care about orientation is taken).
  The claim that the subgroup N would be normally finitely generated is not true in general. It will follow from a suitable finiteness condition on X: $\pi_1(X)$ needs to be finitely presented.
- p14 l-1: "sunsection" should be "subsection"
- p16 top: recall the lower bound on n which is now needed
- p16 l8: "an another" should be "another"

- The following argument, computing the homotopy type of $L_\bullet$ is somewhat vague (I'd say it doesn't really give an argument but just claims the assertion).
There is another spelling mistake: "at the and" should be "at the end".
the displayed equation at the end is an equation in homology; it might be more clear if it was written that way (instead of using a notation for subsets).
- p16 l-2: "to proof" should be "to prove"
- proof of Lemma 7: give more detail
- p17: "a map q lifts to" should be "the map q lifts to"
- more detail at the very end of the proof of Lemma 7: a priori, arguing with simplices showing up at most on one side of the equation of homology classes is risky, as there might be a more complicated chain realizing the equality in homology.
- Remark 2: this deals with a very specific object, "almost equivariant homology", which is not standard knowledge. It would be easier to follow if the definition had been recalled.
- proof of corol 4: "To proof the corollary" should be "To prove the corollary"
- Remark after Corol 4: the transformation from spin-bordism to oriented bordism is a 2-local isomorphism, in particular a rational isomorphism (compare Stong's book). As clearly M_k is zero in rational oriented bordism of $B\pi_1$, any lift of it to (i.e. with any choice of spin structure) to rational spin bordism of $B\pi_1$ is zero, as well. From that point of view, the somewhat convoluted discussion preceeding is not particularly interesting. Of course, one does get somewhat more precise information on the order...