Abstract

To any Gromov hyperbolic group Γ one can associate its boundary—a topological space $\partial\Gamma$ with an action of Γ by homeomorphisms. The boundary of Γ can be endowed with an additional structure of a measure metric space, depending on the choice of a left-invariant hyperbolic metric d on Γ, quasi-isometric to the word metric. The measure μ_d arising from this construction is called the Patterson-Sullivan measure. It is quasi-invariant under the action of Γ, which allows one to define a unitary representation π_d of the group Γ on the space $L^2(\partial\Gamma, \mu_d)$ by the formula

$$[\pi_d(g)f](\xi) = \left[\frac{dg_*\mu_d}{d\mu_d}(\xi)\right]^{1/2} f(g^{-1}\xi).$$

We call it the *boundary representation associated to d*.

The main theorem of this thesis states that the boundary representations are irreducible. This gives an explicit faithful irreducible unitary representation (and in fact, many such representations) of an arbitrary hyperbolic group. It can also be seen as a generalization of the standard fact that the Patterson-Sullivan measures are ergodic. Such result was previously known, due to Bader and Muchnik, in the case where Γ is the fundamental group of a closed negatively curved manifold, endowed with a metric induced by an orbit map of the action of Γ on the universal cover.

The second result of the thesis deals with classification of the boundary representations up to unitary equivalence. The considered family of metrics on Γ can be equipped with a natural equivalence relation of *rough similarity*—two metrics are roughly similar if one of them is similar to a bounded perturbation of the other. The Patterson-Sullivan measures coming from roughly similar metrics are equivalent, and this translates into unitary equivalence of the corresponding boundary representations. We manage to turn this implication into an equivalence, under an additional assumption of double ergodicity of the involved Patterson-Sullivan measures. This assumption is likely to be automatically satisfied—Uri Bader and Alex Furman intend to publish the full proof in their forthcoming paper.