COMPUTER LAB Graphical Models, UWr March 2020

1. **Mathematics Marks of 88 students.**

   Apply the Method $\tilde{K}_{\text{emp}}$ to the table of Mathematics Marks of 88 students and choose the most adapted graphical model.

   (a) Compute K_{emp} and $\tilde{K}_{\text{emp}}$ (EXCEL and R)

   (b) Round up to zero the off-diagonal terms of $\tilde{K}_{\text{emp}}$ such that $|k_{ij}| < 0.1$

   (c) draw the dependence graph

   (d) compare with the results obtained in the Lecture Notes of S. Lauritzen (joint on the last 3 pages of the PART 1 of Lectures Graphical Models, UWr March 2020)

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2. **G-Lasso. Exercise with simulations.**

   (a) Let Γ a symmetric $p \times p$ matrix, with $\Gamma_{ij} = \min(i, j)$.

   Write explicitly Γ for $p = 5$ and $p = 10$.

   (b) Let $e_1, \ldots, e_p$ i.i.d. random variables $N(0, 1)$. Let $Z_k = \sum_{i=1}^{k} e_i$.

   Show that the covariance matrix of the Gaussian vector $Z = (Z_1, \ldots, Z_p)^T$ equals Γ.

   Deduce that Γ is positive definite.

   (c) Compute on the computer $K = \Gamma^{-1}$ for $p = 5$ and $p = 10$.

   Propose $K = \Gamma^{-1}$ for $p \in \mathbb{N}^*$ arbitrary.

   Prove (mathematically, without computer) that $K\Gamma = Id$.

   (d) Give the dependence graph of the Gaussian vector $Z = (Z_1, \ldots, Z_p)$.

   (e) Let $n = 5$ and $p = 10$. Simulate an n-sample A of Z.

   Use $A = rmvnorm(n, rep(0, p), Gamma)$.

   G-Lasso Method of Friedman

   (f) Apply the G-Lasso $glasso(S, rho = ...)$ to the data A.

   For S, take the matrix $var(A)$ of sample covariance of A

   Change the value of the regularisation parameter $\rho = 0.1, 1, 5, 10, 100$.

   The matrix $\tilde{K}$ of conditional correlations is not programed in *glasso*. In order to scale the precision matrix, one can use $cov2cor(wi)$.

   Do you recover the graphical model of Z?

   (g) Apply the G-Lasso *glassopath* to the data A. How does it work?

   (h) Let $n = 50$ and $p = 100$. Simulate an n-sample A of Z.

   Apply *glasso* and *glassopath* to the data A.

   Do you recover the graphical model of Z?

Method of Meinshausen–Bühlmann

(i) Let $n = 5$ and $p = 10$. Simulate an n-sample A of Z.

Apply the Regression Lasso $glmnet(X, Y, alpha = 1)$ to the response variable $Y = A[, i]$ explained by all the other variables $X = A[, -i]$, with $i = 1, \ldots, p$
(One must apply Lasso p times.)
Start by a fixed $i = 1$.

Determine the "best"' value of λ, obtained by crossed validation method: *lambda.min* in $cv.glmnet(X, Y)$. Change $i = 1, \ldots, 10$. Does *lambda.min* depend on i?

Draw the results of Lasso: *plot*. Superpose the vertical line $\lambda = \lambda_{min}$: *abline*

Analyse the coefficients of $glmnet(X, Y, alpha = 1, lambda = \lambda_{min})$

Do you recover the graphical model of Z?

(j) Do (i) for $n = 50$ and $p = 100$. Do you recover the graphical model of Z?

(k) The *glasso* also offers the Method of Meinshausen–Bühlmann,
by $glasso(S, rho = ..., approx = TRUE)$.

Use this option of *glasso*. Compare with (j).

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3. **Graphical Model Selection for Frets' Heads Data.**

Install the Frets' Heads Data table. Use $frets$ in $library(boot)$.

(a) Use the method of $\tilde{K}_{\text{emp}}$ in order to estimate $\tilde{K}$ (here $n = 25 > p = 4$)

(b) Apply the G-Lasso *glasso* with $0.1 \leq \rho \leq 100$ and *glassopath* to Frets' Heads Data.

Round up to 0 the terms $\tilde{k}_{ij}$ of the G-Lasso estimator of scaled precision matrix $\tilde{(K)}$ when $|\tilde{k}_{ij}| < 0.01$.

(c) What graphical model do you select for 4 variables of Frets' Heads, with $0.1 \leq \rho \leq 100$?

(d) What estimator is given by *glasso* for Frets' Heads Data, when $\rho = 0$?

(e) Apply the Meinshausen–Bühlmann method to Frets' Heads Data, with $0.1 \leq \rho \leq 100$.

Validation of COMPUTER LAB in Graphical Models, UWr March 2020

EXERCISES 2 AND 3 (without Exercise 1):

Send a file with scripts, resultats and their interpretation to:
graczyk@univ-angers.fr
before March 25, 2020.

A nice redaction will be appreciated for Frets' Heads Data Ex.3, since the scientific statistical interpretation of Frets' Heads Data is still not accomplished.