Limit Theorems for bm-independent random variables and related topics

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Abstract

The thesis is devoted to the study of some limit theorems for *bm-independent* random variables. The notion of *bm-independence*, introduced in 2010 by Wysoczański, refers to noncommutative random variables which are indexed by elements of a partially ordered set. It generalizes the two universal notions of independence in noncommutative probability: one gets the monotone independence if the index set is totally ordered and the boolean independence if the index set is totally disordered. The random variables we consider, are indexed by elements of the following three classes of positive cones Π in Euclidean spaces $\mathcal{X}$, namely $\Pi = \mathbb{R}_+^d$ in $\mathcal{X} = \mathbb{R}^d$, the *Lorentz cone* $\Pi = \Lambda_d^1$ in $(d+1)-$dimensional Minkowski's spacetime $\mathcal{X} = \mathbb{R}_+ \times \mathbb{R}^d$ and $\Pi = \mathtt{Symm}_d^+(\mathbb{R})$ is the cone of *real symmetric positive definite matrices* in $\mathcal{X} = \mathbb{M}_d(\mathbb{R})$.
In the thesis we study the following two topics:

1. *Analogues of the Law of Small Numbers for bm-independent random variables (bm-LSN)*

2. *Distributions of normalized sums of analogues of Poisson type operators in discrete bm-Fock space.*

Our results show combinatorial descriptions of the limit moments in the language of *bm-ordered noncrossing partitions*.
The first topic of the thesis, the bm-LSN, is a limit theorem formulated for triangular arrays of random variables, indexed by given $\mathbf{I} \subset \Pi$, which are bm-independent and satisfy some normalization conditions in each row. We prove that for each positive cone Π, in the limit, one gets a recursive formula for the moment sequence of a probability measure. It involves a function on noncrossing partitions as well as the *volume characteristic* sequence of the positive cone Π.
The second topic of the thesis deals with the distributions of Poisson type operators. They are constructed out of creation, annihilation and conservation operators on a discrete bm-Fock space. In this case we also find the description of the limit moments by bm-ordered noncrossing partitions with blocks consisting of either one or two elements. Again, our recursive formula for the moments depends on the same function defined on such noncrossing partitions and on the number of singletons in a given partition.